4)

x	$-\infty$	α		0	$+\infty$
$g(x)$	$+$	0	$-$	0	$+$

Partie B

$f(x) = e^{2x} - (x+1)e^x$

1) $* \lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} e^x(e^x - x - 1)$

$= \lim\limits_{x \to +\infty} xe^x(\frac{e^x}{x} - 1 - \frac{1}{x}) = +\infty$

$* \lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} e^{2x} - xe^x - e^x = 0$

2) $f'(x) = 2e^{2x} - [e^x + (x+1)e^x]$

$f'(x) = 2e^{2x} - xe^x - 2e^x$

$f'(x) = e^x[2e^x - x - 2]$

$f'(x) = e^x g(x)$

$e^x > 0$ d'ou $f'(x)$ et $g(x)$ sont de même signe

x	$-\infty$	α		0	$+\infty$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	0	$f(\alpha)$		0	$+\infty$

3) $g(\alpha) = 0 \Rightarrow 2e^\alpha - \alpha - 2 = 0$

$\Rightarrow e^\alpha = \frac{\alpha+2}{2}$ d'où

$f(\alpha) = e^{2\alpha} - (\alpha+1)e^\alpha$

$f(\alpha) = (\frac{\alpha+2}{2})^2 - \frac{(\alpha+1)(\alpha+2)}{2}$

$f(\alpha) = -\frac{(\alpha^2+2\alpha)}{4}$

* soit $h(x) = -\frac{(x^2+2x)}{4}$

$h'(x) = -\frac{1}{2}(x+1) \geq 0$

$\forall x \in [-1,6 ; -1,5]$

h est croissante sur $[-1,6 ; -1,5]$

$(-1,6) \leq \alpha \leq (-1,5)$

$\Rightarrow h(-1,6) \leq h(\alpha) \leq h(-1,5)$

$\Rightarrow 0,16 \leq f(\alpha) \leq 0,185)$

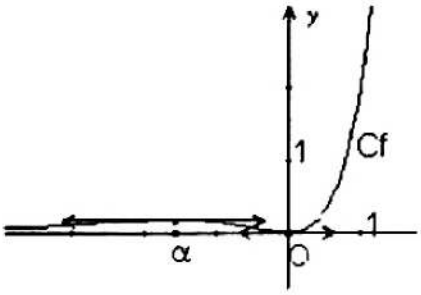
x	$-\infty$	α		0	$+\infty$
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	0	$f(\alpha)$		0	$+\infty$

5) $* \lim\limits_{x \to +\infty} \frac{f(x)}{x} = \lim\limits_{x \to +\infty} \frac{e^{2x}}{x} - (x+1)\frac{e^x}{x}$

$= \lim\limits_{x \to +\infty} \frac{e^x}{x}(e^x - x - 1)$

$= \lim\limits_{x \to +\infty} \frac{e^x}{x^2}(\frac{e^x}{x} - 1 - \frac{1}{x}) = +\infty$

Donc $(\mathcal{C})$ admet une B.I parabolique de direction celle de $(O, \vec{j})$



$\alpha \approx -1,5$

$f(\alpha) \approx 0,17$

6) $m < 0$

a) $\begin{cases} u'(x) = e^x \longrightarrow u(x) = e^x \\ v(x) = x \longrightarrow v'(x) = 1 \end{cases}$

$\int_m^0 xe^x\,dx = [xe^x]_m^0 - \int_m^0 e^x\,dx$

$= -m e^m - [e^x]_m^0 = (1-m)e^m - 1$

b) $\int_m^0 f(x)\,dx = \int_m^0 (e^{2x} - xe^x - e^x)dx$

$= \int_m^0 e^{2x}dx - \int_m^0 xe^x dx - \int_m^0 e^x dx$

$= [\frac{1}{2}e^{2x}]_m^0 - (1-m)e^m + 1 - [e^x]_m^0$

$= \frac{1}{2} - \frac{1}{2}e^{2m} - (1-m)e^m + 1 - 1 + e^m$

$= \frac{1}{2} - \frac{1}{2}e^{2m} + me^m$

c) $\lim\limits_{x \to -\infty} \int_m^0 f(x)\,dx = \frac{1}{2}$

Qarray.tn -Aqra Blech- Qarray.tn